1.1 – Introduction to Systems of Linear Equations

Definition: A **linear equation** can be expressed in the form $a_1x_1+a_2x_2+...+a_nx_n = b$ where a_i are constants, not all zero, and b is a constant.

#14 In each part, use parametric equations to describe the solution set of the linear equation.

a. $x + 10y = 2$

b. $x_1 + 3x_2 - 12x_3 = 3$

c. $4x_1 + 2x_2 + 3x_3 + x_4 = 20$

d. $v + w + x - 5y + 7z = 0$

Definition: A **parameter** (in this context) is a variable that when assigned a value provides a specific solution.

Definitions: A **homogeneous linear equation** is a linear equation in which $b = 0$.

A **system of linear equations** is a finite set of linear equations. It has the form

$$\begin{array}{ccccc} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n & = & b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n & = & b_2 \\ \vdots \qquad \vdots \qquad\qquad \vdots & & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n & = & b_m \end{array}$$

Note that the first of the double subscripts references the equation number, and the second references the term in that equation.

Definition: The **augmented matrix** for the above linear system is

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{bmatrix} \text{ or } \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$$

#16 Each linear system has infinitely many solutions. Use parametric equations to describe its solution set.

a. $$\begin{aligned} 6x_1 + 2x_2 &= -8 \\ 3x_1 + x_2 &= -4 \end{aligned}$$

$$\begin{aligned} 2x - y + 2z &= -4 \\ 6x - 3y + 6z &= -12 \\ -4x + 2y - 4z &= 8 \end{aligned}$$

b.

Definitions: The following are **elementary row operations** performed on a matrix.

1. Multiply a row by a nonzero constant.
2. Interchange two rows.
3. Add a constant times one row to another.

Definitions: A **solution** of a linear system is a sequence of n numbers $s_1,\ s_2,\ \ldots,\ s_n$ that when substituted for corresponding unknowns x_i makes each equation a true statement. If $n = 2$, then the solution is an **ordered pair**, and if $n = 3$, it is an **ordered triple**. In general, an **ordered n-tuple** has the form $(s_1, s_2, \ldots, s_n)$. A linear system is **consistent** if it has at least one solution. A linear system is **inconsistent** if it has no solutions.

#19 Find all values of k for which the given augmented matrix corresponds to a consistent linear system.

a. $\begin{bmatrix} 1 & k & -4 \\ 4 & 8 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 1 & k & -1 \\ 4 & 8 & -4 \end{bmatrix}$

Example: A 3-7-9 diet calls for 3 units of fat, 7 units of protein, and 9 units of carbs in each meal. Suppose an individual has three possible foods to choose from to meet these requirements. Each ounce of the food contains
Food 1: 3 units of fat, 4 units of protein, 1 unit of carbs
Food 2: 2 units of fat, 5 units of protein, 3 units of carbs
Food 3: 4 units of fat, 1 unit of protein, 2 units of carbs
Let x, y, and z denote the number of ounces of the first, second, and third foods that a person will consume at the main meal. Find a linear system in x, y, and z whose solution tells how many ounces of each food must be consumed to meet the diet requirements.